

# Bayesian weak lensing tomography: Reconstructing the 3D large-scale distribution of matter with a lognormal prior

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We present a Bayesian reconstruction algorithm that infers the three-dimensional large-scale matter distribution from the weak gravitational lensing effects measured in the image shapes of galaxies. The algorithm assumes that the prior probability distribution of the matter density is lognormal, in contrast to many existing methods that assume normal (Gaussian) distributed density fields. We compare the reconstruction results for both priors in a suite of increasingly realistic tests on mock data. We find that in cases of high noise levels (i.e. for low source galaxy densities and/or high shape measurement uncertainties), both normal and lognormal priors lead to reconstructions of comparable quality. In the low-noise regime, however, the lognormal model produces significantly better reconstructions than the normal model: The lognormal model 1) enforces non-negative densities, while negative densities are present when a normal prior is employed, 2) better traces the extremal values and the skewness of the true underlying distribution, and 3) yields a higher correlation between the reconstruction and the true density distribution. Hence, the lognormal model is to be preferred over the normal model, in particular since these differences become relevant for data from current and futures surveys.

## I. INTRODUCTION

Weak gravitational lensing of galaxies offers a unique way to study the distribution of matter in the Universe (see [1] for a review on weak gravitational lensing and [2–4] for recent reviews on weak galaxy lensing). Lensing by structures along the line of sight causes distortions in the images of distant galaxies (which in this context are often referred to as sources), which leads to correlations between the apparent shapes of these galaxies. The dominant and most easily detectable image distortion that lensing induces is a shearing of the galaxy images. Because of this, the effect is often referred to as cosmic shear.

Galaxy shape measurements allow to constrain the clustering of matter at different scales and redshifts, which can then be translated into constraints on cosmological models and their parameters. The integrated lensing signal is mostly sensitive to a combination of the cosmic mean matter density  $\Omega_m$  and the matter power spectrum amplitude  $\sigma_8$ . Tomographic methods yield additional constraints on the properties of dark energy and on modified-gravity theories. Since lensing is a direct probe of the total matter, luminous and dark, it can be combined with measurements of the luminous matter distribution in order to learn about the relationship between baryons and dark matter. Another important feature of weak galaxy lensing lies in its ability to probe the matter distribution over a wide range of scales, from many tens of Mega parsec, where structure formation is still linear today and comparably easy to model, down to non-linear sub-Mega parsec scales. Due to its sensitivity to such a wide range of scales, lensing can provide a large amount of information to constrain models of non-linear structure formation and cosmology, in particular if also higher-order statistics are considered.

First firm statistical detections of cosmic shear were reported in 2000 by four different groups [5–8]. Since then, the field has seen a tremendous increase in the amount and quality of lensing data as well as a notable improvement in analysis techniques. A common method to infer cosmological parameters from weak galaxy lensing data is to compare the power spectrum of the fully projected 2D shear field to theoretical predictions. Stronger constraints can be obtained by measuring this power spectrum in a number of redshift bins [9]. This compressed tomographic procedure allows to better assess the time evolution of structure formation and thus to tighten constraints on parameters affecting this evolution. Several authors have investigated the additional constraining power that can be achieved by incorporating third-order statistics and/or shear peak counts and correlations [10–19], which helps to break parameter degeneracies.

The full information content, however, lies in the three-dimensional non-linear shear field. 3D weak shear analysis methods have been proposed by a number of authors [20–22] and were recently applied to data from the Canada France Hawaii Telescope Lensing Survey (CFHTLenS) [23]. Furthermore, the measured shear field can be used for 3D reconstructions of the underlying density field. This can then be directly compared to models of structure formation and simplifies the cross correlation with other tracers of matter. Algorithms that invert the lens equation to obtain the density have been worked out by a number of authors [24–26]. Since this inversion is under-constrained, it requires some regularization method or choice of prior on the density field. Most of the algorithms employ a Wiener

filter, which corresponds to a normal (Gaussian) prior, that can be complemented with information about galaxy clustering [27, 28].

We aim to extend the work on tomographic reconstruction of the 3D matter distribution by designing a fully Bayesian reconstruction algorithm that uses a lognormal prior on the density field. The algorithm is designed to reconstruct the 3D cosmic density fluctuation field  $\delta(\mathbf{x}, \tau)$  from weak galaxy lensing data, i.e. a measurement of galaxy ellipticities at different (photometrically measured) redshifts. Its derivation is based on the language of information field theory [29], which has already been used to address similar tomography problems [30]. We do not make use of the flat-sky approximation i.e. lines of sight are allowed to be non-perpendicular to a fixed 3D coordinate grid. Further, we do not bin the data into pixels but take each galaxy into account as an individual contribution to the likelihood. This allows us to incorporate distance uncertainties of individual galaxies instead of sample redshift distributions.

In contrast to a normal prior for the density field, as it has often been assumed before, a lognormal prior automatically enforces the strict positivity of the field and allows to capture some of the non-Gaussian features that are imprinted on the density distribution by non-linear structure formation. Hubble was the first to notice that galaxy number counts could be well approximated by a lognormal distribution [31]. Characterizing the matter overdensities in the Universe as a lognormal field was first assessed by Coles & Jones in 1991 [32]. Subsequent studies showed that a logarithmic mapping of the nonlinear matter distribution can partly re-gaussianize the field, and that non-linear features in the matter power spectrum can be reproduced by a lognormal transformation of the linear matter power spectrum [33, 34]. A lognormal prior has already been used and shown to be superior to a Gaussian one in Bayesian algorithms that reconstruct the large-scale matter distribution from the observed galaxy distribution [29, 35, 36]. Lognormal distributions have also already been considered in the context of weak lensing: Analyses of ray-tracing simulations and the Dark Energy Survey (DES) Science Verification data showed that the 1-point distribution function of the lensing convergence is better described by a lognormal than a Gaussian model [37, 38]. Also the cosmic shear covariance can be modeled to better accuracy under the assumption that the underlying convergence field follows a lognormal distribution instead of a Gaussian one [39].

Bayesian inference methods are widely used in weak shear analyses, most prominently in the context of shear measurements from galaxy images [40–42]. Recently, notable effort has been put into developing a fully Bayesian analysis pipeline that propagates all uncertainties consistently from the raw image to the inferred cosmological parameters [43, 44].

This paper is organized as follows: This introduction is followed by a short section, Sec. II, in which we briefly introduce the notations and coordinate systems that will be used in the derivation of the formalism. Our lognormal prior model for the density is described in detail in Sec. III. In Sec. IV, we present the data model, i.e. the lensing formalism that connects the data from a cosmic shear measurement to the underlying density field and give a brief overview over its implementation in Sec. V. The maximum a posteriori estimator that is used to infer the matter distribution is introduced in Sec. VI and extended to include redshift uncertainties of individual sources in Sec. VII. In Sec. VIII, we show results of the density reconstruction on increasingly realistic mock data. We conclude this work with a summary and discussion in Sec. IX.

## II. COORDINATE SYSTEMS AND NOTATIONAL CONVENTIONS

In the derivation of the formalism we work with three different types of coordinate systems. First, we use three-dimensional purely spatial comoving coordinates  $\mathbf{x} = (x_0, x_1, x_2)$  at fixed comoving lookback time  $\tau$ , that, combined with the time coordinate, form the four-dimensional coordinate system  $(\mathbf{x}, \tau)$ .

Second, we use a 3D comoving coordinate system on the light cone of an observer at the origin. Vectors on the light cone are marked by a prime, e.g.  $\mathbf{x}'$ . Since  $\mathbf{x}'$ -coordinates implicitly define a comoving lookback time  $\tau(\mathbf{x}') = |\mathbf{x}'|/c$  (where  $c$  denotes the speed of light), we omit spelling out the time explicitly and write  $A(\mathbf{x}') = A(\mathbf{x}', \tau)$  for any quantity  $A$  that is defined on the light cone. The operation that links quantities on the light cone to their corresponding quantities in 4D spacetime can be encoded in a projection operator with kernel

$$C_{\mathbf{x}'(\mathbf{x}, \tau)} = \delta_D(\mathbf{x}' - \mathbf{x}) \delta_D(\tau - |\mathbf{x}|/c). \quad (1)$$

Third, we employ a set of coordinate systems on the light cone, in which each system is orientated such that one axis points into the direction of a source galaxy. These line of sight coordinate systems are centered on the observer and spanned by the vectors  $(\hat{\mathbf{r}}_0^i, \hat{\mathbf{r}}_1^i, \hat{\mathbf{r}}_2^i)$ , where  $\hat{\mathbf{r}}_0^i(\mathbf{x}')$  points into the direction of the  $i$ th source galaxy and the normal vectors  $(\hat{\mathbf{r}}_1^i, \hat{\mathbf{r}}_2^i)$  span the two-dimensional plane perpendicular to  $\hat{\mathbf{r}}_0^i$ . The radial comoving distance of each galaxy from the observer is denoted  $r^i = |\mathbf{r}^i| \equiv |\mathbf{r}_0^i|$ . The transformation from the light cone system into the line of sight (LOS) system of a source galaxy  $i$  is achieved by a rotation of the  $x'_0$ -axis into the  $i$ th line of sight, while  $x'_1$  and  $x'_2$  get aligned with the image coordinates of the galaxy observation. We label the corresponding transformation operators  $\mathcal{R}_{\mathbf{rx}'}^i$ .

### III. PRIOR MODEL

The comoving density  $\rho(\mathbf{x}, \tau)$  can be split into its time-independent spatial mean  $\bar{\rho}$ , and a perturbation  $\delta\rho(\mathbf{x}, \tau) = \bar{\rho}\delta(\mathbf{x}, \tau)$ . The fractional overdensity  $\delta(\mathbf{x}, \tau)$  is commonly modeled as a homogeneous, isotropic Gaussian field with zero mean and power spectrum  $P_\delta(k, \tau)$ . This is an excellent description at early times where fluctuations are very small, as e.g., shown by observations of the cosmic microwave background radiation (CMB). At linear level, valid for  $\delta \ll 1$ , the time evolution of the density field can be described by

$$\rho(\mathbf{x}, \tau) = \bar{\rho}[1 + \delta(\mathbf{x}, \tau)] = \bar{\rho} \left[ 1 + \int_{\mathbf{y}} D(|\mathbf{x} - \mathbf{y}|, \tau, \tau_0) \varphi(\mathbf{y}, \tau_0) \right] = \bar{\rho} [1 + D_{\mathbf{xy}}(\tau, \tau_0) \varphi_{\mathbf{y}}(\tau_0)]. \quad (2)$$

In the last expression we have introduced a short-hand notation that will be used in the rest of the paper: repeated indices are integrated over if they do not appear on both sides of the equation. In Eq. (2),  $D(|\mathbf{x} - \mathbf{y}|, \tau, \tau_0)$  is the integration kernel of a linear homogeneous and isotropic, but possibly scale-dependent, growth operator. The field  $\varphi$  is an isotropic and homogeneous random field whose values are drawn from a multivariate normal distribution with mean  $\bar{\varphi}$  and covariance  $\Phi$ ,

$$\varphi \leftrightarrow \mathcal{N}(\varphi | \bar{\varphi}, \Phi). \quad (3)$$

Here, it describes the three dimensional density fluctuations at time  $\tau_0$  and is translated to other times  $\tau$  by the growth operation  $D_{\mathbf{xy}}(\tau, \tau_0)$ . This implies  $\bar{\varphi} = 0$  for this linear Gaussian model.

The description Eq. (2) breaks down when  $\delta \not\ll 1$  such that non-linearities become important. A possible way to account for non-linearities is to include higher-order terms from a perturbation series expansion of the full non-linear evolution equations. However, a further shortcoming of the model (2) is that it allows arbitrarily negative density contrasts, which physically can not be smaller than -1. To obtain a strictly positive density field, we instead modify Eq. (2) by an exponential:

$$\rho(\mathbf{x}, \tau) = \bar{\rho} [1 + \delta(\mathbf{x}, \tau)] = \bar{\rho} \exp [D_{\mathbf{xy}}(\tau, \tau_0) \varphi_{\mathbf{y}}(\tau_0)]. \quad (4)$$

Since the expectation value of the density  $\rho(\mathbf{x}, \tau)$  must equal  $\bar{\rho}$ , the mean of  $\varphi$  must be set to

$$\bar{\varphi}_{\mathbf{x}}(\tau) = -\frac{1}{2} D_{\mathbf{xy}}^{-1}(\tau, \tau_0) [D(\tau, \tau_0) \Phi(\tau_0) D(\tau, \tau_0)]_{\mathbf{yy}}, \quad (5)$$

for every time  $\tau$ . Note that we integrate over the index  $\mathbf{y}$  in this expression, i.e. the diagonal of the composite operator in square brackets is treated as a field.

For a local growth operator,  $D_{\mathbf{xy}}(\tau, \tau_0) = D(\tau, \tau_0) \delta_{\mathbf{D}}(\mathbf{x} - \mathbf{y})$ , this mean correction simplifies to

$$\bar{\varphi}_{\mathbf{x}}(\tau) = -\frac{1}{2} D(\tau, \tau_0) \hat{\Phi}_{\mathbf{x}}(\tau_0), \quad (6)$$

where we defined  $\hat{\Phi}_{\mathbf{x}}(\tau_0) \equiv \Phi_{\mathbf{xx}}(\tau_0)$ .

The Gaussian field  $\varphi$  and the growth operator  $D$  can be related to known quantities. To see this, consider the expansion of the Fourier modes of  $\delta$  in Eulerian perturbation theory (see e.g. [45]),

$$\delta(\mathbf{k}, \tau) = \sum_{n=1} D^{(n)}(\tau) \delta^{(n)}(\mathbf{k}), \quad (7)$$

where  $\delta^{(n)}$  are convolutions of  $n$  initial fields  $\delta(\mathbf{k}, \tau_0)$  with an integration kernel that changes from order to order. The first term in this series is  $D^{(1)}(\tau)/D^{(1)}(\tau_0) \delta_0(\mathbf{k}, \tau_0)$ , where  $D^{(1)}(\tau)$  is the growing solution to the linearized growth equation [46].

We use this analogy to motivate the simplest possible form of the growth operation in the lognormal model and write

$$D_{\mathbf{xy}}(\tau) = \delta_{\mathbf{D}}(\mathbf{x} - \mathbf{y}) D^{(1)}(\tau) \quad (8)$$

where we have set  $D^{(1)}(\tau_0) = 1$ . This approximation erases any a-priori assumption of scale-dependent growth and mode-coupling of the log field  $\varphi$ . Such a simplification is viable since the model in Eq. (8) describes only our prior assumptions about the density field  $\rho$ . The algorithm will find the most probable realization of  $\varphi$  for a fixed growth operator  $D$  given the data. If a scale-dependence is favored by the data, it will be recovered, at least partially, in the estimate of  $\varphi$ .

Our algorithm also allows to incorporate a more general growth operation at the expense of computation time and memory usage. The application of the most general  $D_{\mathbf{xy}}(\tau)$  generates a four-dimensional field: three-dimensional spatial comoving volumes for every time-slice  $\tau$ . This very large volume is then restricted to a three-dimensional cut by application of the light cone operator [Eq. (1)]. The prior model for the matter overdensity on the light cone then becomes

$$\delta_{\mathbf{x}'} = \mathcal{C}_{\mathbf{x}'(\mathbf{x}, \tau)} \exp [D_{\mathbf{xy}}(\tau) \varphi_{\mathbf{y}}(\tau)] - 1. \quad (9)$$

Applying a complicated, non-diagonal growth operation on all time slices separately before constructing the light cone is numerically not feasible for a high resolution reconstruction. The simplest form of  $D$ , Eq. (8), that depends only on time can be applied to the light cone directly

$$\delta_{\mathbf{x}'} = \exp [D_{\mathbf{x}'\mathbf{y}'} \varphi_{\mathbf{y}'}] - 1. \quad (10)$$

For this work, we use the growth operator in Eq. (8) with the usual linear growth factor and leave extension of this model for future work.

#### IV. DATA MODEL

In this section we establish the analytic relation between the signal field  $\varphi$ , the field of overdensities  $\delta$  that we aim to reconstruct, and the data that is obtained from a weak lensing measurement.

Weak galaxy lensing surveys produce galaxy image ellipticities that can be quantified, e.g., by a complex number  $\epsilon = \epsilon_1 + i\epsilon_2$  such that  $\epsilon = (a - b)/(a + b) \exp(2i\eta)$  for an ellipse with major axis  $a$ , minor axis  $b$ , and position angle  $\eta$ . We use the common approximation that the components of the intrinsic source galaxy ellipticity  $\epsilon^s = (\epsilon_1^s, \epsilon_2^s)$ , which define the shape that would be observed in the absence of lensing<sup>1</sup>, follow a global bivariate Gaussian distribution with zero mean and variance  $\sigma_\epsilon^2$  per component:

$$\epsilon^s \leftarrow \mathcal{N}(\epsilon^s | 0, N^s), \quad N_{ij}^s = \delta_{ij} \sigma_\epsilon^2. \quad (11)$$

This approximation has shortcomings (see e.g. [47]), but serves for the proof of our concept, since we create the mock data on which we test the algorithm with exactly this shape noise model. In the future, more elaborated (Bayesian hierarchical) shear estimators, that e.g. take into account galaxy properties, can be incorporated into the algorithm [40–42].

Lensing distorts the galaxy images in shape and size. If the distortion is small, i.e. in the limit of weak lensing, the relation between intrinsic source ellipticity and observed ellipticity can be linearised and simplifies to

$$\epsilon = g + \epsilon^s, \quad (12)$$

where  $g$  is the reduced shear. The reduced shear combines the effect of anisotropic lensing distortions, encoded in the shear  $\gamma = \gamma_1 + i\gamma_2$ , and the isotropic distortion, encoded in the convergence  $\kappa$

$$g = \frac{\gamma}{1 - \kappa} \approx \gamma. \quad (13)$$

If  $\kappa \ll 1$ , which is often the case for galaxy lensing, the reduced shear can be approximated by the shear itself  $g \approx \gamma$ .

The shear and convergence at angular position  $\boldsymbol{\theta}$  are related to the lensing potential  $\psi$  by

$$\gamma_1(\boldsymbol{\theta}) = \frac{1}{2} (\partial_1^2 - \partial_2^2) \psi(\boldsymbol{\theta}); \quad \gamma_2(\boldsymbol{\theta}) = \partial_1 \partial_2 \psi(\boldsymbol{\theta}); \quad \kappa(\boldsymbol{\theta}) = \frac{1}{2} (\partial_1^2 + \partial_2^2) \psi(\boldsymbol{\theta}). \quad (14)$$

The lensing potential is a weighted projection of the peculiar Newtonian gravitational potential  $\phi$  along the line of sight. For a source at LOS distance  $r^i$ ,

$$\psi(\boldsymbol{\theta}) = \frac{2}{c^2} \int_0^{r^i} dr \frac{r^i - r}{rr^i} \phi(r, r\theta_1, r\theta_2) \quad (15)$$

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<sup>1</sup> We use vector notation, e.g.  $\epsilon$ , to denote the tuple of real and imaginary part  $(\epsilon_1, \epsilon_2)$  of a complex number  $\epsilon = \epsilon_1 + i\epsilon_2$ .

in a spatially flat Universe. Applying the angular derivatives in Eq. (14) to the expression for the lensing potential in Eq. (15), we get

$$\partial_k \partial_l \psi(\boldsymbol{\theta}) = \frac{2}{c^2} \int_0^{r^i} dr W(r; r^i) \partial_{r_k} \partial_{r_l} \phi(r, r\theta_1, r\theta_2), \quad (16)$$

where  $k, l \in (1, 2)$ , and the lensing efficiency

$$W(r; r^i) = \frac{r(r^i - r)}{r^i}. \quad (17)$$

In practice the distance to the source  $r^i$  cannot be determined directly but follows from the photometrically measured redshift  $z^i$ . Photometrically measured redshifts are associated with a relatively high error,  $\sigma_z/(1+z) \approx 0.03$ . In its most simple form the algorithm ignores this uncertainty. We will use this simplified model to validate the functionality of the algorithm in terms of reconstructing non-linear structures in the lognormal approximation. Redshift uncertainties will be included later in Section VII.

The lensing shear is completely determined by the second derivatives of the lensing potential perpendicular to the LOS. The tidal tensor  $\partial_{r_k} \partial_{r_l} \phi(\mathbf{r})$  along the LOS of the  $i$ th source galaxy is obtained by rotating the tidal tensor on the global coordinate grid  $\mathbf{x}'$

$$T_{ij}(\mathbf{x}') = \partial_{x'_i} \partial_{x'_j} \phi(\mathbf{x}'), \quad i, j \in (0, 1, 2), \quad (18)$$

into the specific coordinate system (with coordinates  $\mathbf{r}^i$ ) that points into the direction of this  $i$ th galaxy and projecting it onto the  $(r_1^i - r_2^i)$ -plane perpendicular to the LOS.

The last relation required to connect the data to the density fluctuations is Poisson's equation. It relates the potential  $\phi(\mathbf{x}')$  to the density fluctuations  $\delta(\mathbf{x}')$ ,

$$\nabla^2 \phi(\mathbf{x}') = \frac{3}{2} \Omega_m H_0^2 \frac{\delta(\mathbf{x}')}{a(|\mathbf{x}'|/c)}, \quad (19)$$

where  $H_0$  denotes the Hubble constant (which will be parametrized by  $h = H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1})$  in our test simulations), and  $a(|\mathbf{x}'|/c) = a(\tau)$  denotes the scale factor at the time  $\tau$  corresponding to LOS distance  $r^i(x')$ .

## V. IMPLEMENTATION

The implementation, not only of the data model, but of the entire algorithm, is based on NIFTy [48], a versatile software package for the development of inference algorithms. We further compute cosmology-dependent quantities, like power spectra and distance-redshift relations, with the publicly available CLASS code [49]. To summarize the data model we introduce short-hand notations for each operation in terms of operators.

In its most general form, the prior and data model, that connect the Gaussian field  $\varphi$  with a data vector of  $N_s$  measured source ellipticities, are as follows: The growth operator  $D_{\mathbf{xy}}(\tau)$  imprints a growth structure on the Gaussian field. The resulting four-dimensional field is plugged into the exponential of  $E(\cdot) = \exp(\cdot) - 1$  [see Eq. (4)] to obtain the fractional overdensity  $\delta_{\mathbf{x}}(\tau)$ . The overdensity induces the potential  $\phi_{\mathbf{y}}(\tau)$  by the Poisson equation, encoded in the operator  $\mathcal{P}_{\mathbf{yx}}(\tau) = \Delta^{-1} \frac{3}{2} \Omega_m H_0^2 \delta(\mathbf{x}')/a(|\mathbf{x}'|/c)$  [Eq. (19)]. The potential can be computed efficiently in Fourier space. The gravitational potential is then restricted to the light cone of the observer by the light cone operator  $\mathcal{C}_{\mathbf{y}'(\mathbf{y}, \tau)}$  [Eq. (1)]. We compute the tidal tensor of the resulting 3D field [(Eq. (18))] by application of a global differential operator, which we denote  $\mathcal{T}_{\mathbf{z}'\mathbf{y}'}$ . The resulting tidal tensor is then rotated into each galaxy's LOS coordinate system by a rotation operator,  $\mathcal{R}_{\mathbf{rz}'}^i$ . An integration operator  $\mathcal{I}_{j\mathbf{r}}$ , which applies the integration in Eq. (16), integrates the components of each of the resulting  $N_s$  tidal tensors along the unperturbed photon geodesic. For this operator, we use an adapted version of the implementation that was already successfully used in a similar reconstruction method [30]. The application of  $\mathcal{I}_{j\mathbf{r}}$  yields derivatives of the lensing potential for each galaxy location. From this we can compute the shear components by a linear operator  $\mathcal{L}_{ij}$  that comprises the equations in Eq. (14). Rotation and integration map the three dimensional continuous signal space into the discrete space (one point for every galaxy) of the data. The shear components are thus automatically computed at the locations of the galaxies.

In the simplified implementation that we use for this work, we avoid the 4D coordinate grid  $(\mathbf{x}, \tau)$  and work on the three-dimensional light cone from the beginning. In the prior, we model the Gaussian log-density  $\varphi$  with the power spectrum of matter fluctuations today  $P_{\text{lin}}(k, a=1)$ , where  $a$  denotes the scale factor. The growth operator is

diagonal in configuration space and only a function of comoving distance to the observer  $D_{\mathbf{x}'\mathbf{y}'} = D^{(1)}(\tau)\delta_D(\mathbf{x}' - \mathbf{y}')$ , where  $D^{(1)}(\tau)$  is the growing solution of the linearized growth equation. The Poisson operator is split into two parts. First, a multiplication with  $3/2\Omega_m H_0^2/a(|\mathbf{x}'|/c)$ , i.e. an operation that is diagonal in configuration space. Second, the inverse Laplace operation  $\Delta^{-1}$ , which is diagonal in Fourier space. The inverse Laplacian in the Poisson equation is a non-local operation that should strictly be applied to 3D spatial volumes at fixed time. Here, we apply it on the light cone noting that the induced error in radial direction should be small (roughly of order  $a^2$ ) if we apply the first part of the Poisson operator first and if  $D^{(1)}(\tau)$  is approximately proportional to the scale factor. In this case, the first order term of the exponential expression

$$\exp(D_{\mathbf{x}'\mathbf{y}'}\varphi_{\mathbf{y}'}) - 1 = D_{\mathbf{x}'\mathbf{y}'}\varphi_{\mathbf{y}'} + \mathcal{O}(\varphi^2) \approx a(|\mathbf{x}'|)\delta_D(\mathbf{x}' - \mathbf{y}')\varphi_{\mathbf{y}'} + \mathcal{O}(\varphi^2), \quad (20)$$

and therefore the first order time dependence of the overdensity field will partly be canceled by the  $1/a$  in the Poisson equation before the inverse Laplacian rescales the field. This cancellation corresponds to the commonly known fact that the comoving gravitational potential is constant in a matter dominated Universe.

After computing the six independent entries of the tidal shear tensor, we integrate each of its components along the LOS. Only after the integration we rotate the resulting tensors into their LOS coordinate systems and project out all entries which are not in the plane perpendicular to  $\mathbf{r}$ . This change of order allows to efficiently combine three operations, that is the rotation into the LOS, projecting out non-perpendicular components of the tidal tensor, and the computation of shear components and convergence, in a single linear operation. We denote the corresponding operator  $\mathcal{G}$  (Gamma-Projection-Rotation).

For one data point the implemented data model in operator notation reads

$$d_i = \epsilon_i = (R(\varphi))_i + \epsilon_i^s + \mathbf{n}_i = \mathcal{G}_{ij}\mathcal{I}_{j\mathbf{r}'}\mathcal{T}_{\mathbf{r}'\mathbf{z}'}\mathcal{P}_{\mathbf{z}'\mathbf{y}'}[\exp(D_{\mathbf{y}'\mathbf{x}'}\varphi_{\mathbf{x}'}) - 1_{\mathbf{y}'}] + \epsilon_i^s + \mathbf{n}_i. \quad (21)$$

The total data vector  $d$  has dimensions  $2 \times N_s$ , i.e. two ellipticity components for each of the  $N_s$  source galaxies. We use the letter  $R$  to encode the total response of the data to a signal  $\varphi$ , i.e. the composite action of all operators. We have also added an experimental noise  $\mathbf{n}$  here for completeness. In general this will be subdominant to the shape noise  $\epsilon^s$  and we ignore it in the following. Note, however, that the formalism allows for an incorporation of several independent noise sources.

## VI. MAP ESTIMATOR

Our aim is to obtain a maximum a posteriori (MAP) estimate of the signal field  $\varphi$ . The posterior distribution is related to the likelihood and the prior by Bayes' theorem

$$\mathcal{P}(\varphi|d) = \frac{\mathcal{P}(d|\varphi)\mathcal{P}(\varphi)}{\mathcal{P}(d)}. \quad (22)$$

The prior probability  $\mathcal{P}(\varphi)$  is modeled as a Gaussian distribution with covariance  $\Phi$ . In a first simple approximation  $\Phi$  can be taken to be diagonal in Fourier space with the usual power spectrum of matter overdensities  $P_{\text{lin}}(k, a)$ .

To obtain the likelihood, we marginalize over all realizations of the shape noise

$$\mathcal{P}(d|\varphi) = \int \mathcal{D}\epsilon^s \mathcal{P}(d|\varphi, \epsilon^s) \mathcal{P}(\epsilon^s) = \mathcal{N}[d|R(\varphi), N], \quad (23)$$

where the covariance  $N$  of the shape noise was defined in Eq. (11) and we neglect any other sources of measurement noise. With this, the negative log-posterior becomes

$$-\ln \mathcal{P}(\varphi|d) \hat{=} \frac{1}{2} [d - R(\varphi)]^\dagger N^{-1} [d - R(\varphi)] + \frac{1}{2} (\varphi - \bar{\varphi})^\dagger \Phi^{-1} (\varphi - \bar{\varphi}), \quad (24)$$

where we have dropped most terms that are independent of the field of interest,  $\varphi$ .

The maximum of the posterior distribution is found by minimizing the expression in Eq. (24). Note that the posterior distribution is not Gaussian. Due to the exponential in the response  $R$  it is not quadratic in  $\varphi$ . To find the minimum of the negative log-posterior we apply a Newton-like minimization scheme [50]. This requires the derivative of the negative log-posterior with respect to  $\varphi$

$$-\frac{\delta \ln \mathcal{P}(\varphi|d)}{\delta \varphi_{\mathbf{u}}} = \Phi_{\mathbf{u}\mathbf{q}}^{-1} (\varphi - \bar{\varphi})_{\mathbf{q}} + [d - R(\varphi)]_i N_{ij}^{-1} \mathcal{G}_{jk} \mathcal{I}_{k\mathbf{r}'} \mathcal{T}_{\mathbf{r}'\mathbf{z}'} \mathcal{P}_{\mathbf{z}'\mathbf{y}'} [\exp(D_{\mathbf{y}'\mathbf{x}'}\varphi_{\mathbf{x}'}) * D_{\mathbf{y}'\mathbf{u}}], \quad (25)$$

where the star denotes a pointwise product in position space, i.e.  $(\alpha * \beta)_{\mathbf{x}} = \alpha_{\mathbf{x}}\beta_{\mathbf{x}}$ .

## VII. REDSHIFT-MARGINALIZED LIKELIHOOD AND POSTERIOR

We can take into account the source redshift uncertainty by extending the marginalized likelihood in Eq. (23) by the redshift distribution function  $\mathcal{P}(z^s)$

$$\mathcal{P}(d|\varphi) = \int \mathcal{D}n \int \mathcal{D}z^s \mathcal{P}(d|\varphi, n, z^s) \mathcal{P}(n) \mathcal{P}(z^s) = \int \mathcal{D}z^s \mathcal{N}[d|R^{z^s}(\varphi), N] \mathcal{P}(z^s), \quad (26)$$

where  $z^s$  denotes the vector of redshifts of all sources. In most cases, this integration cannot be done analytically and the resulting distribution will in general not be Gaussian. However, to good approximation we can fit the marginalized likelihood with a Gaussian that we characterize by the first and second central moment of the full distribution.

To obtain them, we need to calculate  $\langle d \rangle_{\mathcal{P}(d|\varphi)}$  and  $\langle dd^\dagger \rangle_{\mathcal{P}(d|\varphi)}$ . For the expectation value of the data, we obtain

$$\langle d \rangle_{\mathcal{P}(d|\varphi)} = \int \mathcal{D}z^s \int \mathcal{D}d d \mathcal{N}[d|R^{z^s}(\varphi), N] \mathcal{P}(z^s) = \int \mathcal{D}z^s R^{z^s}(\varphi) \mathcal{P}(z^s) \quad (27)$$

$$\begin{aligned} \langle d_i \rangle_{\mathcal{P}(d|\varphi)} &= \mathcal{G}_{ij} \left[ \int \mathcal{D}z^s \mathcal{P}(z^s) \mathcal{I}_{j\mathbf{r}}^{z^s} \right] \bar{R}(\varphi)_{\mathbf{r}} = \mathcal{G}_{ij} \left[ \int_0^\infty dr \int_r^\infty dr^j W(r, r^j) \mathcal{P}(z_j^s) \frac{dz_j^s}{dr^j} \right] \bar{R}(\varphi)_{\mathbf{r}} \\ &\equiv [\mathcal{G} \bar{\mathcal{I}} \bar{R}] (\varphi)_i \equiv \tilde{R}(\varphi)_i, \end{aligned} \quad (28)$$

where  $\bar{\mathcal{I}}$  denotes the redshift averaged integration operator defined in the square brackets in the first line of Eq. (28) and we have introduced  $\tilde{R}(\cdot) \equiv \mathcal{TP}(\exp[D(\cdot)] - 1)$  to summarize the action of all source-redshift independent operators.

The second moment is

$$\langle dd^\dagger \rangle_{\mathcal{P}(d|\varphi)} = N + \langle (R(\varphi))(R(\varphi))^\dagger \rangle_{\mathcal{P}(z)}. \quad (29)$$

Non-diagonal elements of the second term in Eq. (29) read

$$\langle ((R(\varphi))_i (R(\varphi))_j) \rangle_{\mathcal{P}(z)} = \langle (R(\varphi))_i \rangle_{\mathcal{P}(z)} \langle (R(\varphi))_j \rangle_{\mathcal{P}(z)} = (\tilde{R}\varphi)_i (\tilde{R}\varphi)_j \quad (30)$$

and diagonal elements are

$$\begin{aligned} \langle ((R(\varphi))_i (R(\varphi))_i) \rangle_{\mathcal{P}(z)} &= \mathcal{G}_{ij} \mathcal{G}_{ij} \left[ \int_0^\infty dr \int_0^\infty dr' \int_{\max(r, r')}^\infty dr^j W(r, r^j) W(r', r^j) \mathcal{P}(z_j^s) \frac{dz_j^s}{dr^j} \right] \bar{R}(\varphi)_{\mathbf{r}} \bar{R}(\varphi)_{\mathbf{r}'} \\ &\equiv \mathcal{G}_{ij} \mathcal{G}_{ij} \tilde{\mathcal{I}}_{j\mathbf{r}, \mathbf{r}'} \bar{R}(\varphi)_{\mathbf{r}} \bar{R}(\varphi)_{\mathbf{r}'}, \end{aligned} \quad (31)$$

where the new operator  $\tilde{\mathcal{I}}$  denotes the squared average of the integration operator, i.e. the square brackets in the first line of Eq. (31).

The Gaussian approximation to the likelihood is then  $\mathcal{N}[d|\tilde{R}(\varphi), \tilde{N}]$ , where  $\tilde{N} = N + Q$  and  $Q$  is

$$Q = \langle (R(\varphi))_i (R(\varphi))_j \rangle_{\mathcal{P}(z)} - \langle (\tilde{R}\varphi)_i \rangle_{\mathcal{P}(z)} \langle (\tilde{R}\varphi)_j \rangle_{\mathcal{P}(z)} = \langle (R(\varphi))_i (R(\varphi))_i \rangle_{\mathcal{P}(z)} - (\tilde{R}\varphi)_i (\tilde{R}\varphi)_i. \quad (32)$$

This expression is still signal-dependent and we approximate it further by replacing  $\varphi$  by its posterior mean  $\langle \varphi \rangle_{\mathcal{P}(\varphi|d)}$ . Since this mean depends on  $\tilde{N}$ , the resulting set of equations must be solved iteratively.

## VIII. VALIDATION AND TESTS

To validate the implementation and assess the goodness of the tomographic reconstruction, we perform a number of increasingly realistic tests, which we denote tests A, B and C. In tests of type A we employ an even source distribution over the entire box. These tests are the least realistic ones and serve to validate the correctness of the implementation. Test B uses a realistic source distribution and test C adds realistic shape noise.

In all of these tests, we place the observer in the center of the bottom of the computational box<sup>2</sup> and resolve the underlying and reconstructed overdensity fields with  $128^3$  pixels. Depending on the test, we allow the physical sizes

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<sup>2</sup> This position is not fixed by the algorithm. For a reconstruction from a survey that covers a significant fraction of the sky, the observer can be placed in the center of the box, for example.

TABLE I. Quantitative comparison of reconstruction methods for tests A2 and A3. We compare the mean square pixel-wise difference between the underlying and reconstructed fields ( $\delta$  and  $\hat{\delta}$ , respectively), their minimal and maximal values, and the Pearson-correlation coefficient between the true density and the reconstructions.

redshifts	$\langle(\hat{\delta} - \delta)^2\rangle/\sigma_\delta^2$		$\max(\hat{\delta}), \max(\delta)$			$\min(\hat{\delta}), \min(\delta)$			$\sqrt{\langle\hat{\delta}\delta\rangle}/(\sigma_\delta\sigma_{\hat{\delta}})$	
	lognormal	WF	lognormal	WF	N-body	lognormal	WF	N-body	lognormal	WF
1.	0.70	0.71	4.07	1.69	4.92	-0.64	-1.04	-0.79	0.56	0.54
0.25	0.67	0.72	7.30	2.05	8.49	-0.87	-5.02	-0.87	0.58	0.54
0.0	0.66	0.71	7.83	2.27	10.26	-1.00	-5.96	-0.89	0.59	0.55
light cone	0.71	0.83	33.03	3.56	70.20	-1.00	-12.03	-0.92	0.56	0.44

of the box to differ. The current pixel resolution is limited due to computation time and memory usage. A higher resolution will be accessible after parallelization and adaption of the code for the usage on a high-performance cluster.

Most of the tests are based on mock data that we create by applying the data model described in Sec. IV to non-linear density fields obtained from N-body simulations. For tests at fixed redshifts we use snapshots of the Millennium-XXL simulation [51] from which we take sub-volumes of size  $[1h^{-1}\text{Gpc}]^3$  and smooth the density fields to fit the desired resolution of  $128^3$  pixels. For more realistic test cases we construct a light cone of size  $[500h^{-1}\text{Mpc}, 500h^{-1}\text{Mpc}, 4000h^{-1}\text{Mpc}]$  by joining snapshots of a sub-volume of the Millennium Run [52]. This sub-volume measures  $500h^{-1}\text{Mpc}$  along each side and we shift it every time  $500h^{-1}\text{Mpc}$  in the  $z$ -direction have been constructed in order to avoid that a LOS hits the same structure repeatedly. In the resulting light cone we achieve a resolution of  $3.9h^{-1}\text{Mpc}$  in the  $x$ - and  $y$ -directions corresponding to mildly non-linear scales and reach a redshift of  $z = 2.2$  in the  $z$ -direction. Since we use  $128^3$  pixel to resolve the light cone box, the physical size of the pixels is longer along the  $z$ -axis, meaning that we get a poorer resolution in this direction.

Both the Millennium-XXL and the Millennium simulation, use a flat  $\Lambda\text{CDM}$  cosmology with  $\Omega_m = 0.25$  and  $h = 0.73$ .

### A. Tests A: Simple geometry

In this series of tests we distribute 500 000 sources evenly in the box and slightly beyond ( $\pm 100h^{-1}\text{Mpc}$ ). We place sources outside of the reconstructed volume in order to increase the area in which we can recover the underlying density field with high resolution. The quality of the reconstruction decreases with increasing distance to the observer because of increasing distances between lines of sight and a decreasing number of background sources that help to break the LOS degeneracy of the lensing kernel. We further only add unphysically low, negligible shape noise in this first test set.

We perform three different tests of type A for which we create data from increasingly realistic input fields:

1) A self-consistency test, where we create an overdensity field from a random realization of the Gaussian field  $\varphi$  [Eq. (3)] in a cubic box of side length  $1000h^{-1}\text{Mpc}$ . We then apply the implemented data model and use the algorithm to recover this input field. We do not show any results from this test, since they would not provide any additional insights compared to tests A2 and A3, but simply state that we can recover the input field with high fidelity which means that the implementation is in itself consistent.

2) A test in which we create shears from overdensity fields taken from an N-body simulation at fixed redshifts. We use three different snapshots at individual redshifts  $z = 1$ ,  $z = 0.25$  and  $z = 0$ . This allows to assess the algorithm's ability to recover increasingly non-linear fields. In each case, we compare the lognormal reconstruction to a Wiener filter (WF) reconstruction, which uses the same data model but a Gaussian prior on the overdensity field  $\delta$ . Comparisons between input and reconstructed fields in each case are shown in Fig. 1. Table I summarizes pixelwise quadratic differences between underlying and reconstructed fields, minimal and maximal values and the pixel-wise Pearson correlation between the true density and its reconstructions. In all these parameters and for all redshifts tested the lognormal prior yields better results than the Gaussian prior. The difference between lognormal and WF reconstruction increases as the input field becomes more non-linear. This is also reflected in the 1-point probability distribution functions (PDFs), which we show in Fig. 2. While the Wiener Filter PDF is almost symmetric in all cases, we can capture some of the skewness of the input field by applying the lognormal prior. An notable feature of the 1-point PDFs is that the maximum value is biased in both reconstructions. The distributions of the reconstructions peak at zero, i.e. at their mean, while the underlying field peaks below. This is a feature of both priors since they prefer the mean density (or a zero density contrast) if the data does not contain enough information on the density.



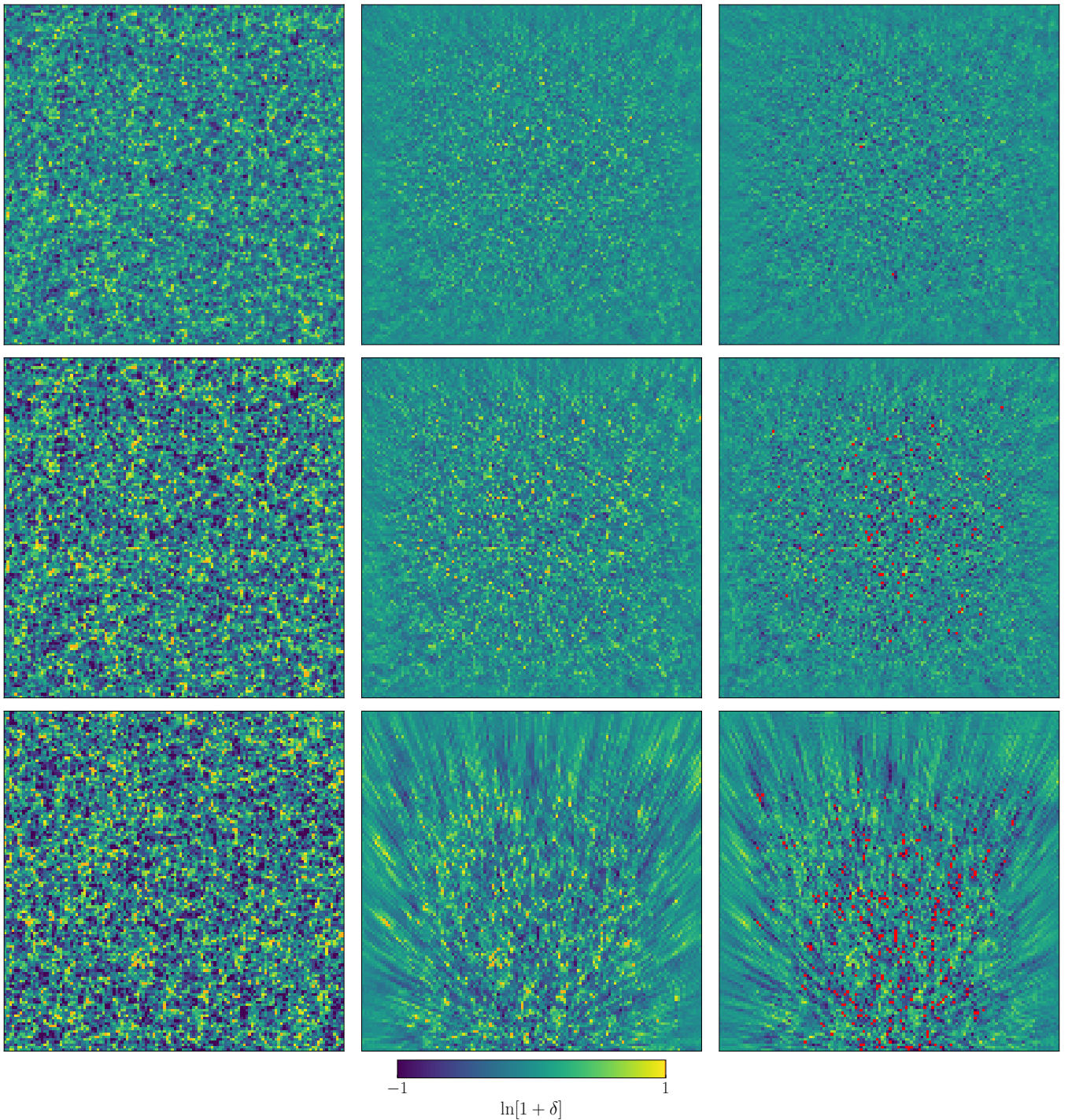


FIG. 1. Qualitative comparison of reconstruction methods in the test series A2, for which we create mock data from individual simulation snapshots, apply an even source distribution and add negligible shape noise. We show central slices through the 3-dimensional fields. The observer is located in the center of the  $(x - y)$ -plane at the origin of the  $z$ -axis. The redshift of the snapshot decreases from top to bottom,  $z = [1, 0.25, 0.]$ . In the two upper rows we show the  $(x - y)$  plane at  $z = \max(z)/2$ , in the last row we look at the  $(x - z)$ -plane at  $y = \max(y)/2$ . The fields are from left to right: the underlying density field from the simulation, its reconstruction with a lognormal prior and its reconstruction with a Gaussian prior (Wiener filter). We plot  $\ln[1 + \delta]$  and mark unphysical negative densities in the Wiener Filter reconstruction in red. The resolution of the reconstruction decreases with distance to the observer. This is because 1) the density of lines of sight decreases 2) there are less sources behind the point we want to reconstruct 3) the information from these sources is suppressed by the shape of the integration kernel. In all cases, the lognormal reconstruction is superior in capturing the highest values of the density field and avoids unphysically low density contrasts below -1.

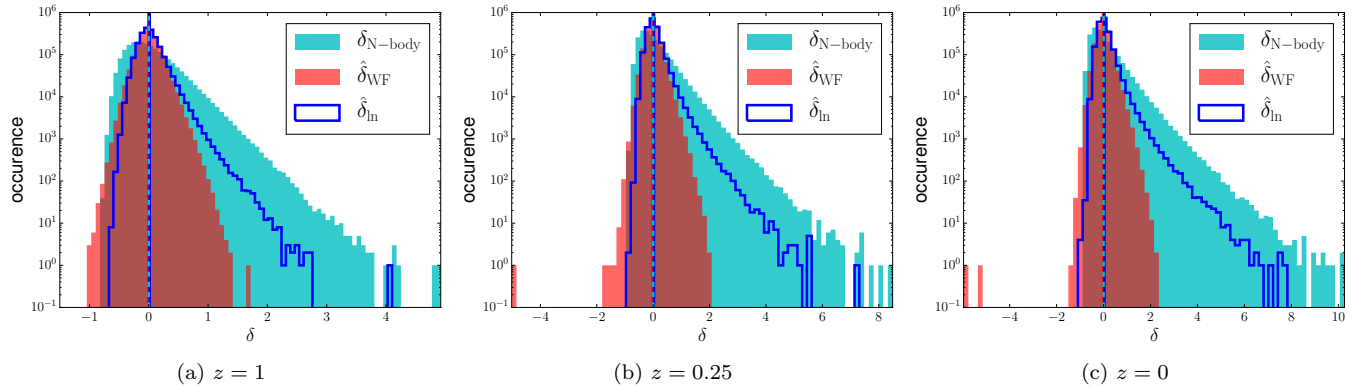


FIG. 2. 1-point PDFs of the simulated density contrasts (in cyan) and their Wiener filter and lognormal reconstructions (red and dark blue) in tests on mock data created from different snapshots of an N-body simulation with an even source distribution and negligible shape noise (A2). The PDF of the lognormal reconstruction is slightly more skewed than the Wiener filter PDF, which is close to symmetric in all cases. Both the Wiener filter and the lognormal reconstruction are equally biased in the position of the peak, however, the mean values of both reconstructions agree well with the mean value of the underlying density. The mean values of the density contrast are indicated by vertical lines in the same color as their corresponding distribution. We also find that the Wiener filter produces unphysical density contrasts below -1 in all snapshots.

3) A test similar to A2 but with a redshift dependent density field constructed from different snapshots. The physical size of the box spans  $[500h^{-1} \text{ Mpc}]^2$  in the  $(x-y)$ -plane and  $4000h^{-1} \text{ Mpc}$  into the  $z$ -direction corresponding to a maximum redshift of  $z = 2.2$ .<sup>3</sup> Results of test A3 are shown in Fig. 3 and Fig. 4. The quantitative comparison between the lognormal and the corresponding WF run is listed in the last row of Tab. I labeled “light cone”. We find a similar match between the underlying and reconstructed fields as in tests A2 and again a superiority of the lognormal prior over the WF reconstruction. This superiority is still visible if we smooth the results of both reconstructions and the underlying field with a Gaussian kernel with  $\sigma = 8h^{-1} \text{ Mpc}$  (Fig. 3, second row).

### B. Test B: Realistic survey geometry

In the test of type B, we employ a realistic survey geometry and source distribution. We use input overdensities from the same light cone as in test A3 to generate mock data and place sources in a cone that spans  $7.15^\circ$ . The sources are distributed according to a distribution function of form

$$n(z) = z^\alpha \exp(-(z/z_0)^\beta), \quad (33)$$

where we fit  $\alpha$ ,  $\beta$ , and  $z_0$  to the publicly available source distribution of the CFHTLenS survey<sup>4</sup>. The resulting source distribution function is shown in the left panel of Fig. 5. The fit does not exactly match the survey distribution, but the similarity is sufficient for this test. We then draw  $\Omega[\text{arcmin}^2]\rho_s[\text{gal/arcmin}^2]$  source positions from this distribution, where  $\Omega$  is the angular opening area of the cone in  $\text{arcmin}^2$  and  $\rho_s$  the source density of the survey, which we choose to be  $\rho_s = 11 \text{ gal/arcmin}^2$  corresponding to the source density in CFHTLenS. A scatter plot of the resulting spatial source distribution is shown in the right panel of Fig. 5. The reconstructions are depicted in Fig. 6.

We find that the superiority of the lognormal model remains with a realistic background galaxy distribution. If we restrict our analysis to the region that is covered by the observational cone, we find that the distribution of the reconstructed field values closely follows the underlying density distribution if we apply the lognormal prior. The Gaussian prior fails to capture the skewness and produces negative densities (Fig. 7).

<sup>3</sup> The light cone is constructed by merging planes from different snapshots along the  $z$ -direction of the box, but the algorithm assumes that the distance and redshift increase in radial direction from the observer. This leads to a mismatch between the distance to a (source) position assumed by the algorithm and the distance or rather redshift that this position corresponds to in the simulation. The distance in the simulation is  $\cos \theta$  times the distance assumed by the algorithm, where  $\theta$  is the angle between the LOS and the  $z$ -axis. This mismatch should only become relevant for relatively large angles  $\theta > \pi/2$ . Lines of sight with such angles only cover a minor fraction of the box and we therefore expect the test to not be severely affected by this approximation.

<sup>4</sup> <http://www.cfhtlens.org/astronomers/cosmological-data-products>

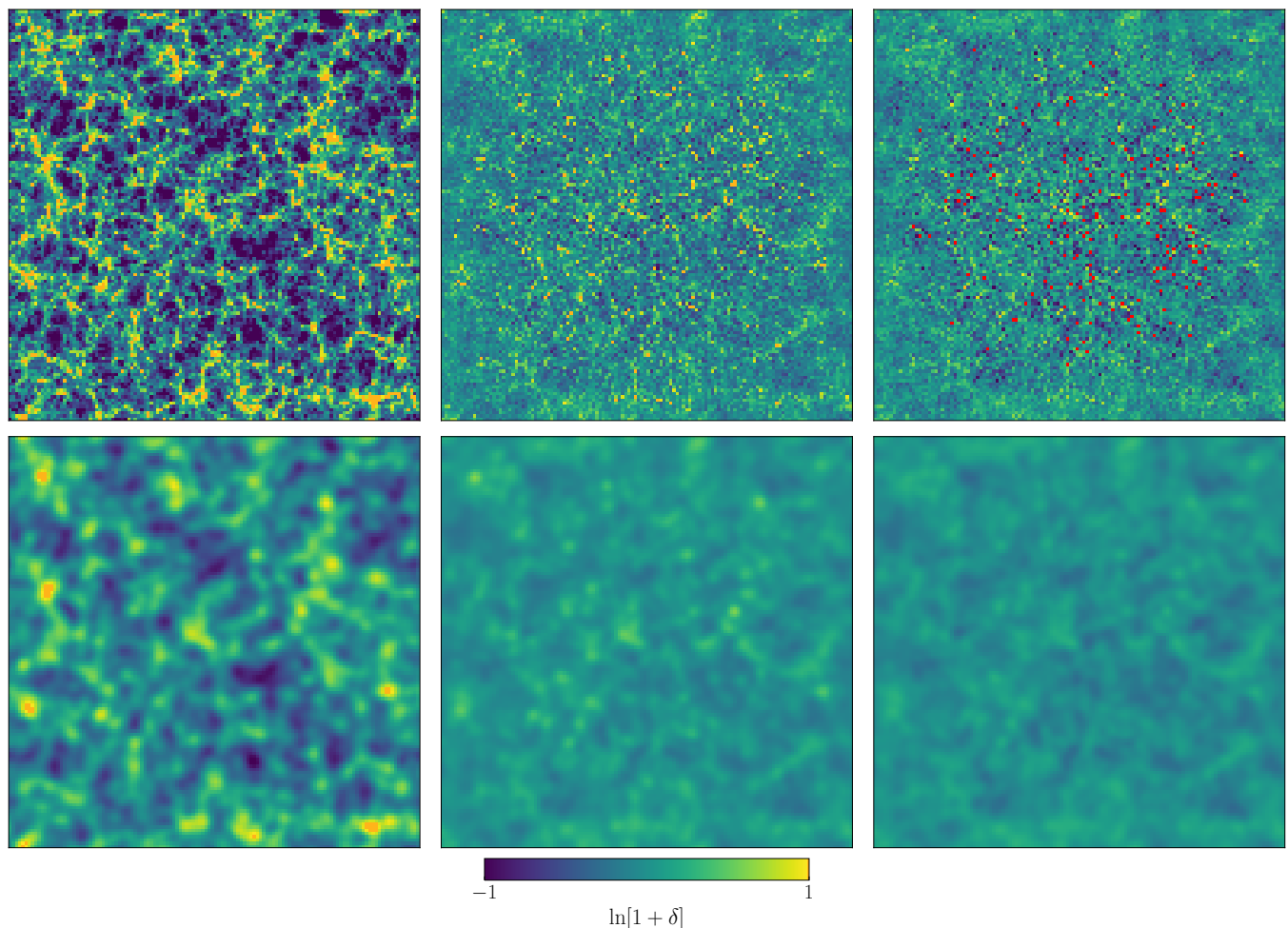


FIG. 3. Qualitative comparison of reconstruction methods in test A 3. For this test, we create mock data from a simulated density field on a light cone, use an even source distribution and add negligible shape noise. The fields are from left to right: the underlying overdensity field from the simulation, the reconstructed overdensity field using a lognormal prior and the reconstructed overdensity using a Gaussian prior (Wiener filter). We show the central  $(x - y)$ -plane of the reconstruction box in each panel. The observer is located in the center of the  $(x - y)$ -plane at the origin of the  $z$ -axis. Note that we plot  $\ln[1 + \delta]$  and mark negative densities in red. In the second row, we show a smoothed version of the fields. The smoothing is performed with a Gaussian kernel with  $\sigma = 8h^{-1}$  Mpc.

### C. Test C: Realistic shape noise

In the last test of our test series, we add realistic shape noise with variance  $\sigma_\epsilon^2 = 0.25$  to the mock data on the light cone. This value is close to the value in CFHTLenS,  $\sigma_\epsilon^2 = 0.279$  [53]. Compared to test B, we also extend the opening angle to  $10.0^\circ$  and use a slightly higher galaxy density of  $15 \text{ gal/arcmin}^2$ . We show results of this final test in Fig. 8. The test reveals that the lognormal model does not yield any significant improvement over a Wiener Filter reconstruction for mock data that is similar to CFHTLenS data. The data has only constraining power on scales where the density field is still well in the Gaussian regime. Smaller scales could become accessible with higher source densities, such as current and future surveys will provide. Tests with such higher resolutions will be possible after upgrading the code for the usage on a high performance cluster.

## IX. CONCLUSIONS

We have presented a novel, fully Bayesian, lensing tomography algorithm that reconstructs the three-dimensional matter distribution from measurements of individual galaxy shapes. The main difference to existing lensing recon-

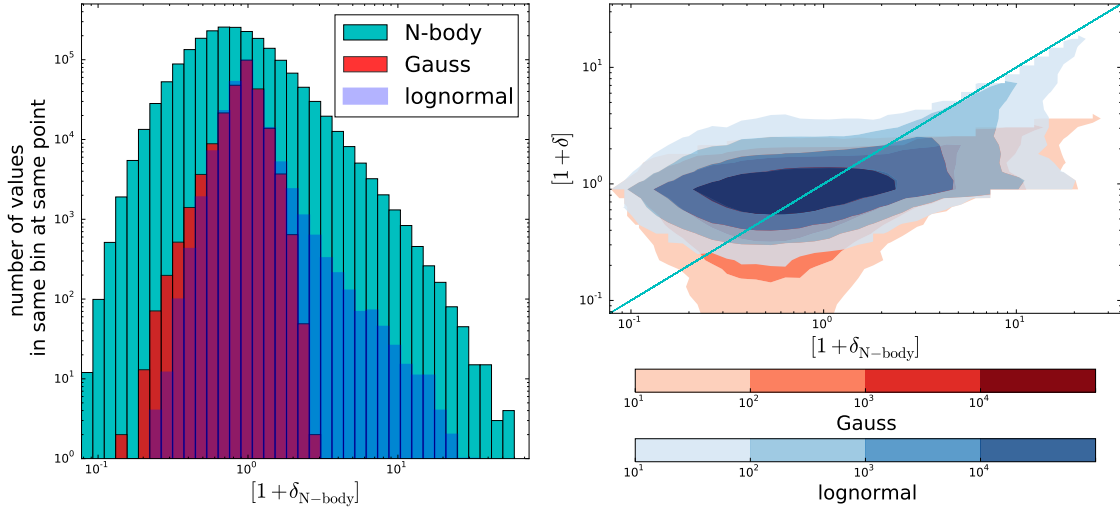


FIG. 4. Comparison of reconstruction methods in test A3: In the left panel, we have binned the underlying density into 40 bins of equal width (0.17) in  $\ln$ -space and count the points in the reconstructions whose density lies in the same bin at the same location. The total number of points in a density bin in the original density field is shown for comparison in the background (cyan). This plot shows that the lognormal model is better in capturing overdensities, while it is only slightly worse than the WF reconstruction in capturing underdensities (note the log scaling). In the right panel, we plot the full distributions of the reconstructions against the underlying density. A perfect reconstruction would follow the diagonal. The lognormal reconstruction achieves this better than the Gaussian one, given that the data is sufficiently constraining. At points that are poorly constrained, both reconstructions prefer values around the horizontal, which corresponds to the mean density.

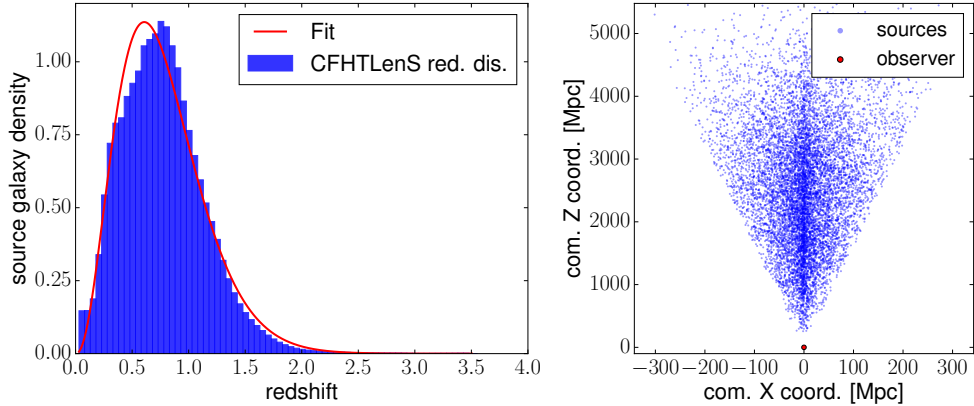


FIG. 5. The source redshift distribution used for test B: In the left panel, we show the histogram of source redshifts from CFHTLenS and our fit to this distribution of the form given in Eq. (33). The right panel depicts the projected 3D scatter plot of the galaxy sample that is drawn from the fit and used in test B (we only show every 200th source).

struction methods lies in the use of a lognormal prior on the density field instead of a Gaussian one. The lognormal model is an improvement over the Gaussian approximation in two ways: First, it enforces the strict positivity of the density while a Gaussian prior allows unphysical, negative densities. Second, it incorporates an a priori knowledge of the presence of odd moments in the matter distribution, which arise as a consequence of non-linear structure formation. These corrections are relevant since the cosmic shear signal probes structures down to scales that lie well in the non-linear regime. We note that the lognormal distribution does not describe the exact distribution of non-linearly evolved overdensities. However, since it is only used as a prior, it will be updated by the information contained in the data. In regions with high signal-to-noise, this prior model only supports the high-fidelity reconstruction. In less well constrained regions it prevents unphysical features in the reconstructed field.

To assess the differences between a reconstruction with a lognormal and a Gaussian prior (the latter corresponds to a Wiener filter), we have applied the algorithm with both priors to increasingly realistic mock data sets. Those data

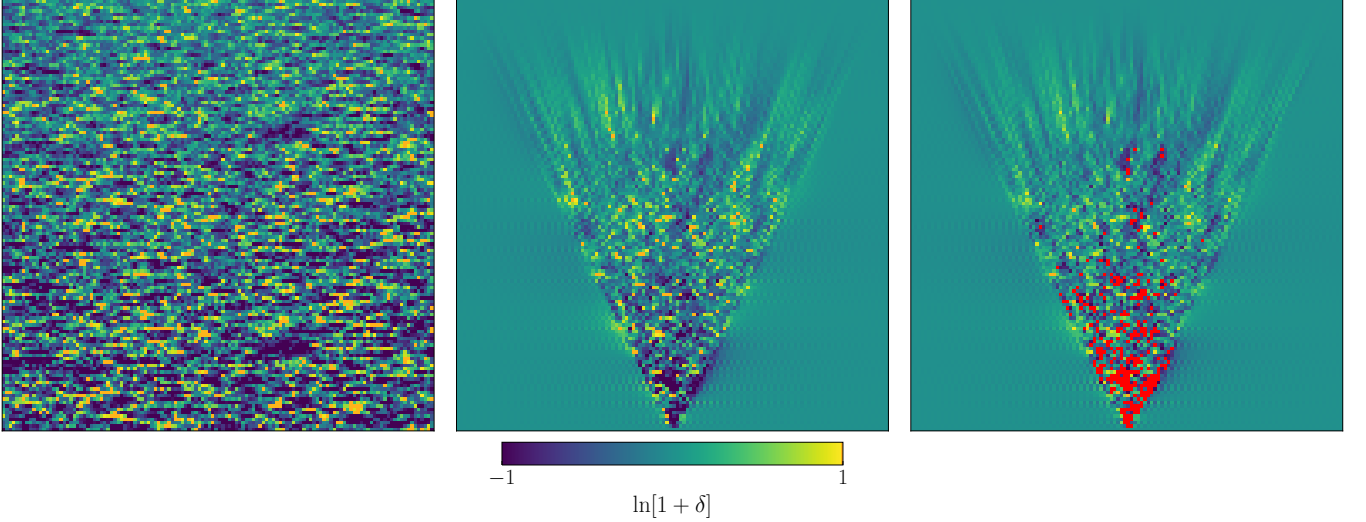


FIG. 6. Results of test B, reconstructions of a redshift-dependent density field on the observer’s light cone from mock data with a realistic source distribution but negligible shape noise. We show central slices through the  $y$ -axis of the underlying field,  $\ln[\delta_{\text{N-body}} + 1]$ , in the left, the lognormal reconstruction,  $\ln[\hat{\delta}_{\text{ln}} + 1]$ , in the middle and the Wiener Filter reconstruction,  $\ln[\hat{\delta}_{\text{WF}} + 1]$  in the right panel. Negative densities are marked in red. The apparent anisotropy of the fields stems from the fact that we have squeezed them in the  $z$ -direction. We use square pixels in the plot, while their physical size is eight times larger in the  $z$ -direction than in the  $x$ -direction.

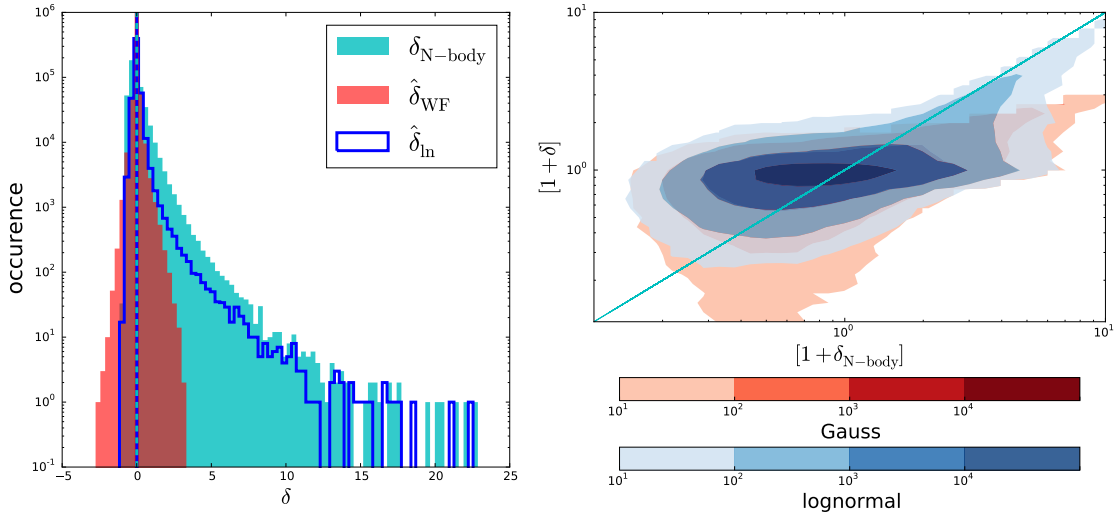


FIG. 7. Results of test B, reconstructions from mock data with a CFHTLenS-like source distribution. We have restricted the analysis to the field values that lie within the observed cone. In the left panel we show the 1-point PDFs of the fields with mean values indicated by vertical lines in the same color as their corresponding distribution. The right plot shows the density distributions of the reconstructions against the underlying density. The lognormal model is clearly better in tracing the highest values of the true density distribution.

sets were produced by applying the algorithm’s inherent data model to non-linearly evolved densities from N-body simulations. In tests with negligible shape noise, but a realistic CFHTLenS-like source distribution, we find that our reconstruction method is superior to the Wiener filter: It avoids negative densities that are present in the Wiener filter reconstructions. It is better in reconstructing the highest local peaks in the density field, and it leads to a higher pointwise correlation between the true underlying density and its reconstruction. We also find that the lognormal model yields no significant improvement over the Wiener Filter when we reconstruct from CFHTLenS-like mock data with realistic shape noise. This might, however, change for data from current and future surveys, that have a higher

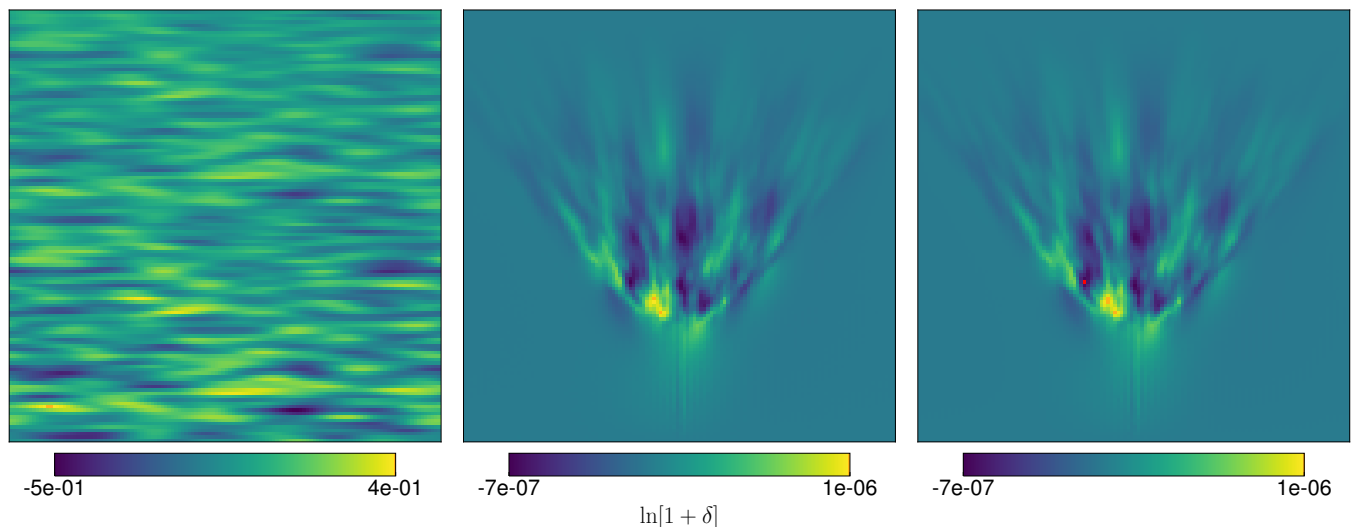


FIG. 8. Results of test C: reconstructions from CFHTLenS-like mock data with realistic shape noise. In the left panel we show a smoothed version of the underlying density field on a non-central slice through the  $y$ -axis. The central and middle panel depict the corresponding slices through the lognormal and WF reconstructions, respectively. The test reveals that current data is only poorly constraining on the smaller scales, where the advantages of the lognormal model would become relevant. For the analysis of CFHTLenS data we find no improvement over the existing Wiener Filter method.

source density and more constraining power on smaller scales.

The Bayesian formulation also allows for an easy and consistent integration of other sources of information or uncertainties. In this work we show how photometric redshift uncertainties can be included, but leave the implementation for future work. The algorithm can further be complemented with models of intrinsic alignments or extended to use the clustering of galaxies as an additional source of information on the underlying matter distribution. Finally, it can be combined with existing Bayesian methods for shear estimation from galaxy surveys, yielding a complete Bayesian analysis pipeline. The outcome of an improved 3D reconstruction can then, for example, be used for measurements of the cross-correlation with other tracers of matter or for studying higher order field statistics.

In this paper, we have proven that the algorithm yields better density reconstructions than existing methods in idealized test cases. In the future, a parallelized implementation of the algorithm, that can handle larger data sets, should be tested on realistic mock data of current and future surveys. These tests will reveal for which survey characteristics this new algorithm can achieve a significant improvement over existing methods.

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